

== 定積分の部分積分法, 置換積分法 ==

○ 定積分の部分積分法

定積分の部分積分法は, 次の公式によって行う.

$$\int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

ただし, $f'(x), g'(x)$ は各々 $f(x), g(x)$ の導関数

例

$$\int_0^{\frac{\pi}{2}} x \sin x dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx$$

x を微分すると 1 になる (次数が下がる) ことに着目する.

微分する側	$f(x) = x$	→	$f'(x) = 1$
積分する側	$g(x) = -\cos x$	←	$g'(x) = \sin x$

$$\text{(原式)} = [-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= (0 - 0) + [\sin x]_0^{\frac{\pi}{2}} = 1$$

短答問題

次の定積分を求めよ. (入力は半角数字…計算用紙: 必要)

(1) $\int_1^e x \log x dx = \dots = \frac{e^2 + \square}{\square}$

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(2) $\int_1^e \log x dx = \dots = \square$

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(3) $\int_0^1 x e^x dx = \dots = \square$

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(4) $\int_1^3 (x-1)^2(x-3)dx = \dots = -\frac{\square}{\square}$

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○ 定積分の置換積分法

定積分の置換積分法では,

(1) 被積分関数

$$\int_a^b f(x) dx$$

(2) 積分変数

$$\int_a^b f(x)dx$$

(3) 積分区間

$$\int_a^b f(x)dx$$

の3箇所を書き換える.

※定積分は, 積分区間の下端・上端の値を代入すると定数になるので, 不定積分の置換積分法とは異なり, 変数を元に戻す必要はない.

例

$$\int_1^2 (2x-1)^3 dx$$

$2x-1=t$ とおく

$$\frac{dt}{dx}=2 \rightarrow dx = \frac{dt}{2}$$

$$\begin{array}{c|c} x & 1 \rightarrow 2 \\ \hline t & 1 \rightarrow 3 \end{array}$$

$$\text{(原式)} \int_1^3 t^3 \cdot \frac{dt}{2} = \frac{1}{2} \left[\frac{t^4}{4} \right]_1^3 = \frac{1}{2} \left(\frac{81}{4} - \frac{1}{4} \right) = 10$$

例と答

(1)

$$\int_0^1 \sqrt{1-x} dx$$

$1-x=t$ とおく

$$\sqrt{1-x} = t^{\frac{1}{2}}$$

$$\frac{dt}{dx} = -1 \rightarrow dx = -dt$$

$$\begin{array}{c|c} x & 0 \rightarrow 1 \\ \hline t & 1 \rightarrow 0 \end{array}$$

$$\text{(原式)} = \int_1^0 t^{\frac{1}{2}} (-dt) = \int_0^1 t^{\frac{1}{2}} dt$$

$$= \left[\frac{2}{3} t^{\frac{3}{2}} \right]_0^1 = \frac{2}{3}$$

(2)

$$\int_0^3 \sqrt{9-x^2} dx$$

$x=3 \sin t$ とおく

$$\begin{array}{c|c} x & 0 \rightarrow 3 \\ \hline t & 0 \rightarrow \frac{\pi}{2} \end{array}$$

$$\sqrt{9-x^2} = \sqrt{9-9\sin^2 t} = \sqrt{9(1-\sin^2 t)}$$

$$= 3 \cos t \quad (0 \leq t \leq \frac{\pi}{2} \text{ のとき } \cos t \geq 0)$$

$$\frac{dx}{dt} = 3 \cos t \rightarrow dx = 3 \cos t dt$$

$$\text{(原式)} = \int_0^{\frac{\pi}{2}} 3 \cos t \cdot 3 \cos t dt$$

$$\int_{\frac{\pi}{2}}$$

$$= 9 \int_0^{\frac{\pi}{2}} \cos^2 t dt$$

$$= 9 \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2t}{2} dt = \frac{9}{2} \left[t + \frac{\sin 2t}{2} \right]_0^{\frac{\pi}{2}} = \frac{9\pi}{4}$$

(3)

$$\int_0^1 2x(x^2+1)^3 dx$$

$x^2+1=t$ とおく

$$\frac{dt}{dx} = 2x \rightarrow dx = \frac{dt}{2x}$$

x	$0 \rightarrow 1$
t	$1 \rightarrow 2$

(原式) $\int_1^2 2x t^3 \frac{dt}{2x} = \int_1^2 t^3 dt$

$$= \left[\frac{t^4}{4} \right]_1^2 = \frac{15}{4}$$

(4)

$$\int_0^1 \frac{1}{1+x^2} dx$$

$x = \tan t$ とおく

x	$0 \rightarrow 1$
t	$0 \rightarrow \frac{\pi}{4}$

$$\frac{1}{1+x^2} = \frac{1}{1+\tan^2 t} = \cos^2 t$$

$$\frac{dx}{dt} = \frac{1}{\cos^2 t} \rightarrow dx = \frac{dt}{\cos^2 t}$$

(原式) $\int_0^{\frac{\pi}{4}} \cos^2 t \cdot \frac{1}{\cos^2 t} dt = \int_0^{\frac{\pi}{4}} dt$

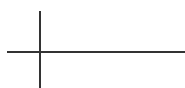
$$= \left[t \right]_0^{\frac{\pi}{4}} = \frac{\pi}{4}$$

短答問題

次の定積分を求めよ。(入力は半角数字…計算用紙: 必要)

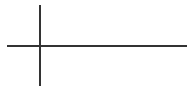
(1) $\int_0^3 \sqrt{3-x} dx = \dots = \square \sqrt{\square}$

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(2) $\int_0^5 \sqrt{25-x^2} dx = \dots = \frac{\boxed{}}{\boxed{}} \pi$

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(3) $\int_1^e \frac{\log x}{x} dx = \dots = \frac{1}{\boxed{}}$

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(4) $\int_0^2 \frac{dx}{4+x^2} = \dots = \frac{\pi}{\boxed{}}$

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